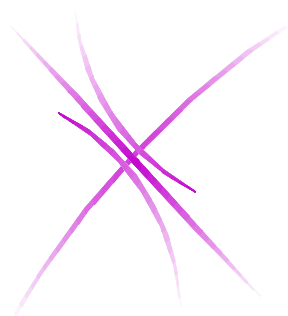


**The Relationship between  
Sustainable Growth and the Risk-  
free Rate: Evidence from UK  
Government Gilts**

*Staff Working Paper 2012.1*

Andrew Lilico & Stefano Ficco  
January 2012



**Europe Economics**

**STAFF WORKING PAPER**



## FOREWORD

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The present paper has been prepared by Dr Andrew Lilico & Dr Stefano Ficco. Andrew is a Principal and Stefano a Senior Consultant at Europe Economics. The paper explores why one would expect the sustainable growth rate of the economy to be related to the risk-free rate, and explores the implications for the relationship between the average growth rate of the economy and the yield on government gilts.

It is hoped that this work will provide a relevant technical contribution to an important area of current debate.

### For further information contact:

**Europe Economics  
Chancery House  
53-64 Chancery Lane  
London WC2A 1QU  
Tel: (+44) (0) 20 7831 4717  
Fax: (+44) (0) 20 7831 4515  
E-mail: [requests@europe-economics.com](mailto:requests@europe-economics.com)  
[www.europe-economics.com](http://www.europe-economics.com)**

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# **The Relationship between Sustainable Growth and Risk-free Rate: Evidence from UK Government Gilts**

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# The Relationship between Sustainable Growth and Risk-free Rate: Evidence from UK Government Gilts

*This paper investigates whether movements in index-linked government bond yields are correlated with movements in medium-term GDP growth rates in the way one might expect from theory. The paper finds that movements in UK ten-year index-linked gilts can now be seen to have been highly correlated with movements in average ten-year GDP growth (a relationship that has not been obvious until the recent recession). As theory implies should be possible, we appear to be able to infer an excellent forecast for the ten-year ahead growth rate of the economy from the yield on index-linked bonds. When we apply this model to current data, the forecast growth rate is markedly lower than that produced by approaches such as those used by the UK Office for Budget Responsibility.*

## 1 Introduction

The purpose of this paper is to test whether movements in index-linked government bond yields are correlated with movements in medium-term GDP growth rates in the way one might expect, given the theoretical relationship that exists between risk-free rates of return on investment and the sustainable growth rate of the economy. We test this relationship using UK index-linked gilts data.

The statistical relationship we identify can be used to infer a forwards-looking sustainable growth rate for the economy directly from market data. This contrasts with standard techniques for estimating sustainable growth rates from cyclical analysis. Conversely, alternative evidence on medium-term growth rates could be used to infer estimates for current and future risk-free rates on a forwards-looking basis, as an alternative to techniques that focus upon historical averages.

The remaining of the paper is organised as follows: Section 2 sets out the related literature. Section 3 provides an overview of the theoretical relationship between market returns and macroeconomic conditions, and between sustainable growth and risk-free rate. Section 4 describes the data used and the statistical relationship found. Section 5 concludes.

### 1.1 Related Literature

Our study here should be understood in the context of three key strands of literature:

- First, economic growth models, such as those in the Ramsey-Cass-Koopmans tradition, in which a key equilibrium condition is that (absent population growth) the sustainable growth rate of the economy equals the risk-free rate.<sup>1</sup>
- Second, corporate finance theories relating the cost of capital (or elements thereof) to economic growth — for example, variants of the Gordon growth model.<sup>2</sup>

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<sup>1</sup> Ramsey, F.P. (1928), "A mathematical theory of saving", *Economic Journal*, 38, 152, pp543–559. Cass, D. (1965), "Optimum Growth in an Aggregative Model of Capital Accumulation", *Review of Economic Studies*, 37 (3), pp233–240.

<sup>2</sup> Gordon, M.J. (1959), "Dividends, Earnings and Stock Prices". *Review of Economics and Statistics*, 41 (2), pp99–105.



- Standard practitioner techniques for estimating the sustainable growth rate and output gap from cyclical analysis, such as those deployed by the UK Office for Budget Responsibility or the Bank of England.<sup>3</sup>

## 2 Theoretical context

We shall begin by setting out various theoretical reasons one might expect movements in risk-free rates to be correlated with movements in the sustainable growth rate of GDP. We emphasize that when, in later sections, we conduct a statistical investigation of this relationship, we believe that should be understood as verifying whether a relationship that is strongly grounded in theory is also evidenced in data. We are not claiming to establish the existence of the relationship statistically — that is established by the theory that follows. The statistics merely confirm that we find what we are expecting to find, given the theory.

### 2.1 *Impact of the economy on total returns to enterprises*

When economic growth is higher, firms tend to have greater earnings. Demand is higher, so the gross value added by businesses increases. Faster economic growth leads to greater total enterprise returns.

So, if economic growth is expected to be higher in the future, there are expected to be greater enterprise returns. Total enterprise returns are divided between labour and capital. If the split (the ratio) can be taken as given (or indeed if returns to labour can be taken as fixed), then a rosier economic outlook implies that returns to capital will be greater. If investors, responding to a rosier economic outlook, did not demand higher returns, they would be conceding that labour would take all the benefit from faster growth. Normally, however, capital demands its share of the expected larger pie.

This is the straightforward case, but it is worth noting that there is no iron rule here. If there is a change in the capital/labour split of returns, that could in principle reverse the overall effect or enhance it. For example, poor economic times could coincide with a fall in the share of total returns taken by labour, so that total returns to capital could rise even as total enterprise returns fell — in which case our straightforward case effect would be reversed. As an alternative example, rosier economic times could coincide with labour taking a lower share of total returns — so our straightforward case effect would be enhanced.

If a period of elevated returns is relatively brief — for example, if it occurs only for a year or two in the recovery phase from a recession — then although actual returns to capital may be higher, the required rate of return will not. Over the lifetime of an investment, there will naturally be some years in which actual rates of return are below the cost of capital and others in which actual rates of return are higher. But overall, average expected rates of return will equal the cost of capital.

On the other hand, periods of slower or higher growth could be more sustained than this. In economics, the “long-term” refers to the period over which there are no fixed costs — when all investments must be renewed. A period of low or high growth sustained for a longer period than the lifetime of investments is not merely cyclical in nature; it is structural, and should be expected to affect not merely year-to-year actual returns but also the required rate on return

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<sup>3</sup> See, for example, *Estimating the output gap*, Briefing Paper No. 2, Office for Budget Responsibility, April 2011 — <http://budgetresponsibility.independent.gov.uk/wordpress/docs/briefing%20paper%20No2%20FINAL.pdf> and Pybus, T. *Estimating the UK's historical output gap*, Working Paper No. 1, Office for Budget Responsibility, November 2011



on investment, because if low / high growth is sustained and economy-wide, then it affects the opportunity cost of investment; we can neither invest in something else nor can we simply wait a brief time and invest under more favourable circumstances.

We observe that economic “shocks” affecting the sustainable growth rate can be both good and bad in nature. There might be new technologies that raise the sustainable growth rate (e.g. by stimulating more rapid innovation); there might be periods of sustained bad weather damaging harvests (e.g. for a couple of decades).

## **2.2 Relationship between the sustainable growth rate and the risk-free rate**

It is common to think of the risk-free rate of return as an exogenous taste variable — if not actually constant, then at least fixed by factors outside portfolio decision-making. We think of the risk-free rate as a measure of impatience, of how much we would rather have things today than tomorrow.

However, though there is much in this, it is not quite the whole story. For the risk-free rate is not simply the return any one individual would require to hold a risk-free asset. Rather, it is the return that would be available in such an asset. As such, (a) it reflects collective tastes, rather than those of any individual — the “taste” of the Market; and (b) it reflects an (albeit notional) equilibrium condition.

In standard long-term economic growth models, such as the Ramsey-Cass-Koopmans model, a key equilibrium condition is that (absent population growth) the sustainable growth rate of the economy equals the risk-free rate.<sup>4</sup> Indeed, in corporate finance theory the risk-free rate of return is sometimes viewed as arising from the sustainable growth rate (i.e. causality runs from the sustainable growth rate to the risk-free rate).

For our purposes here, we need not fully endorse either of these positions. Instead, we make the more limited claim that one should expect both that there is some relationship between the levels of the risk-free rate and the growth rate (albeit not necessarily identity) and that, over a sufficiently long timescale, changes in the risk-free rate to be correlated with changes in the sustainable growth-rate.

We can make this thought more concrete by considering the likely relationship between the sustainable growth-rate and our best proxy for the risk-free rate, namely yields on index-linked government bonds. If, for example, yields on medium- to long-term government bonds are very low, we should interpret that as an indicator that the sustainable growth rate of the economy is expected to be very low. Why? Well, consider an investor that is willing to buy a government bond at a very low yield. That investor is choosing to purchase that government bond in preference to, for example, shares or bonds in any other business in the real economy. But that must indicate that expected returns for the real assets of these other real economy businesses are expected to be low or very volatile. Let us set aside the high volatility case for now, and focus on the case in which returns of these real economy businesses are low. If returns to all real assets are low, over the medium- to long-term, then the economy can only be expected to grow slowly over the medium- to long-term. But the sustainable growth rate is simply the rate at which the economy can grow over the medium- to long-term. So (setting aside issues of policy mistakes etc. that might eventually be rectified), when government bond yields are very low, one plausible explanation is that the sustainable growth rate of the economy is very low.

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<sup>4</sup> Ramsey, F.P. (1928), “A mathematical theory of saving”, *Economic Journal*, 38, 152, pp543–559. Cass, D. (1965), “Optimum Growth in an Aggregative Model of Capital Accumulation”, *Review of Economic Studies*, 37 (3), pp233–240.



We observe that, if international capital markets were perfect and perfectly integrated (e.g. there were no segmentation), then the arguments above would relate the world sustainable growth rate to the world risk-free rate. The implication would then be that to focus on the relationship within one country would either assume (a) that that country's trade and other exposures to international events were such that that country's sustainable growth rate were the same as the world's sustainable growth rate or least that there were a very high correlation between the average growth rate in that one country and the sustainable growth rate of world GDP; or (b) that capital markets were segmented, such that there were a separate risk-free rate for the country of interest. For our purposes here we shall assume that either (a) or (b) is true of the UK.

### **2.3 Implication**

The implication we draw from the above is that, as a broad generalisation, one should expect there to be a relationship between movements in the sustainable growth rate of GDP and the risk-free rate of return, and hence between index-linked government bond yields and average growth rates over the subsequent period.

## **3 Data and Statistical Relationship**

In order to establish whether a statistical relationship exists between growth and risk-free rates we have considered the following data:

- GDP
- Yields on UK index-linked government bonds (index-linked gilts, or ILGs)

### **3.1 Why index-linked gilts?**

We note that the first index-linked gilts were not issued until 1981, and the Bloomberg data series we employ in this study commences only in 1985. That means that, until the 2008/9 recession, index-linked gilts data only spanned one recession — that of the early 1990s. By contrast, UK nominal government bond yields data can be obtained that go back centuries. So why are we focused upon index-linked gilts?

One problem with the use of nominal bond yields is how to convert them into expected real returns. For that, one would need to infer inflation expectations. Obtaining robust estimates of inflation expectations is not straightforward – indeed, one of the best sources is the difference between nominal and index-linked bond yields (obviously not relevant in this case). There are techniques available, based on rational expectations models (and others).<sup>5</sup>

We do not dispute the interest of such techniques. But the use of index-linked data circumvents the need to make such adjustments. Furthermore, the index-linked data is of particular interest since it is the standard starting point, in UK regulatory determinations of the cost of capital, for estimates of the risk-free rate — nominal government bond data serves only as a cross-check.<sup>6</sup> If movements in index-linked gilts can be shown to be related to movements in

<sup>5</sup> One class of such techniques is deployed in papers such as Shiller, R.J. (1981) "Do stock prices move too much to be justified by subsequent changes in dividends?" *American Economic Review*, 71, pp421-436.

<sup>6</sup> e.g. see [http://www.ofwat.gov.uk/pricereview/pr09phase3/rpt\\_com\\_20091126fdcc.pdf](http://www.ofwat.gov.uk/pricereview/pr09phase3/rpt_com_20091126fdcc.pdf), especially Section 2 and Appendix 1.



the sustainable growth rate, then it might be possible to predict future movements in the risk-free rate from estimates of changes in the sustainable growth rate. The exercise is thus of interest both in respect of implications for the forecasting/analysis of the sustainable growth rate (and related issues such as the output gap and the cyclically-adjusted government deficit) from index-linked gilts data and also for the forecasting/analysis of the risk-free rate from sustainable growth rate data.

### **3.2 *Why only the UK?***

Whilst relationships in the UK data, and their implications for risk-free rate and macroeconomic analysis have their own interest, one would expect fundamental economic relationships (such as those we posit between risk-free rate and the sustainable growth rate) to pertain in other countries, also.

A number of other countries' governments do issue index-linked bonds — in particular Canada, France, Sweden and the US. Unfortunately, Canada, France, and Sweden do not have consistent series going back ten years or more (indeed, the Canadian data series only covers 20+ year index-linked bonds). The US 10-year index-linked bond series goes back to 1997, but as such covers only the current recession and thus is insufficient for our purpose.

Only the UK data are sufficiently long-standing. Indeed, even in the case of the UK data, they have only recently come to have covered two recessions. Thus we believe we are amongst the first analysts to investigate the particular question of this paper using these data.

### **3.3 *Relationships in the UK ILG data***

In Figure 1, we illustrate the data-series for ten-year gilts. We observe two general points. First, the series exhibits a clear downward trend. Second, there are two significant drops in the series: one in 1992; and one from the late 1990s. The fall in yields of the late 1990s was interpreted by a number of authorities at the time as indicating some distortion to the market — perhaps associated with pension rules, in particular.<sup>7</sup>

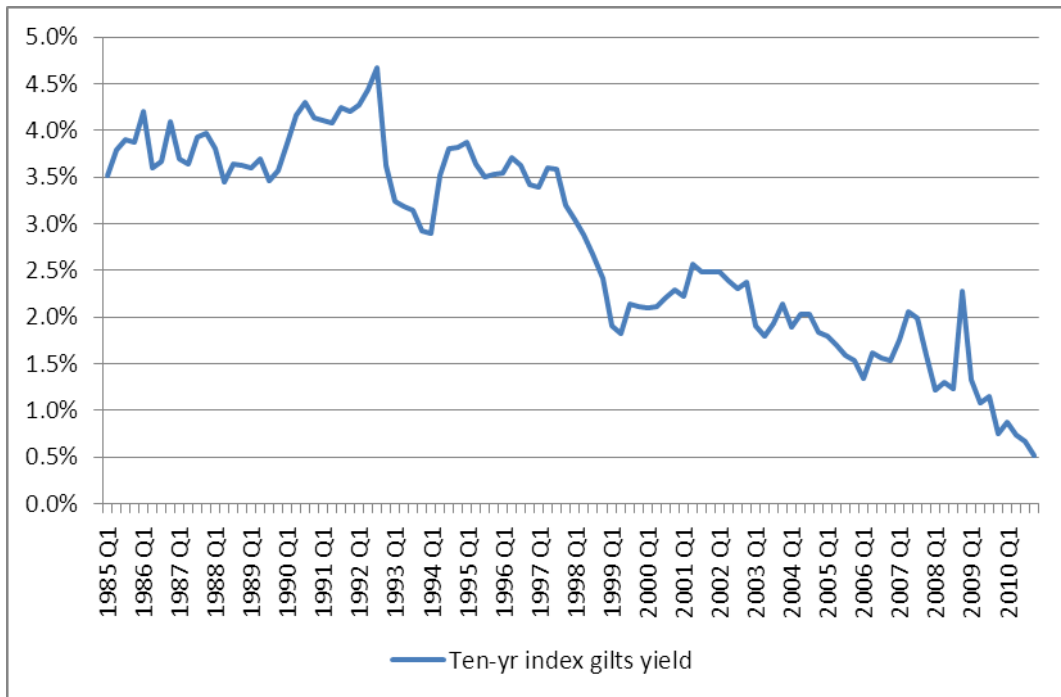
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<sup>7</sup> For example, see Bank of England, 1999, Quarterly Bulletin, May.





**Figure 1: Ten-year index gilts yield**

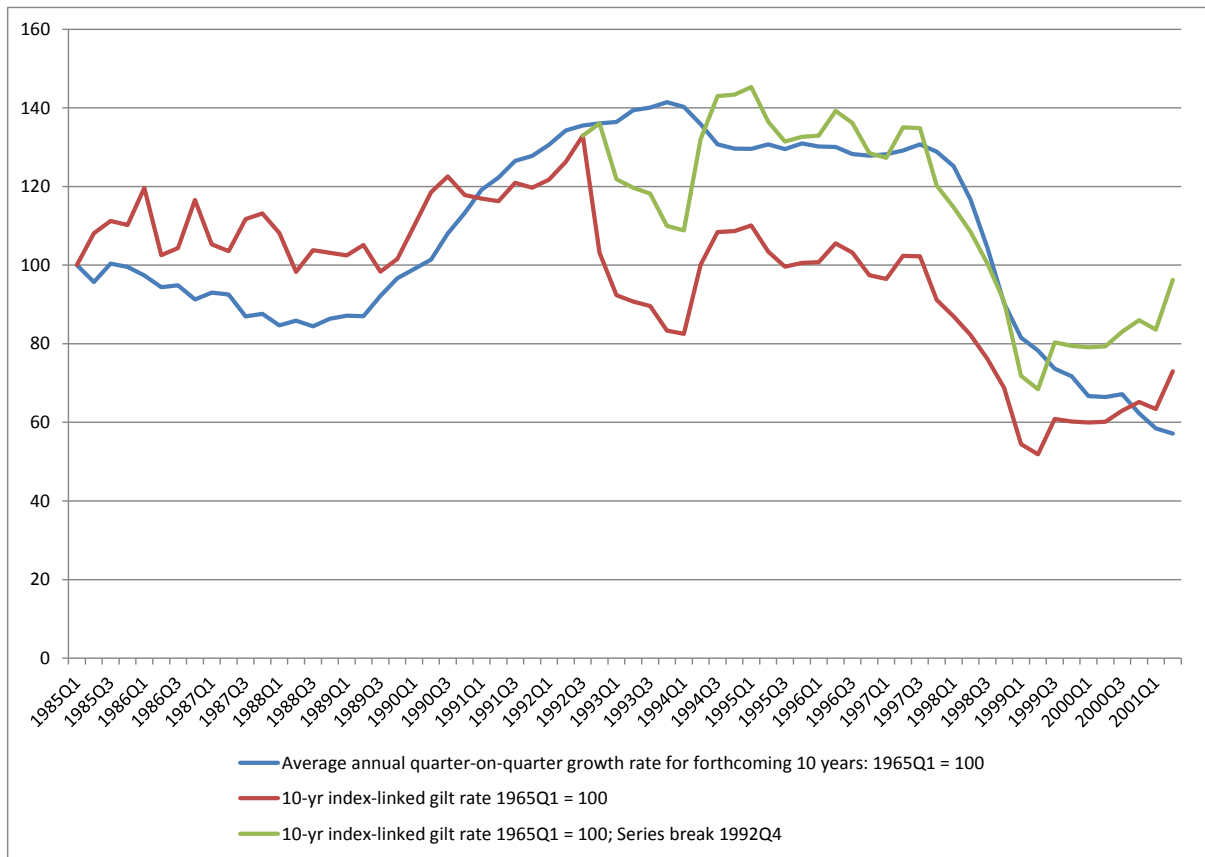


Next, in Figure 2 we compare the average quarterly yield on ten-year index-linked bonds (in blue) with the actual average growth rate over the subsequent ten years (in red). Note that the GDP growth number we use is the geometric average growth in quarterly GDP on that same quarter ten years earlier. We are not using annual GDP growth data.

To make the relationship easier to see, we have normalised both series so that, as they begin in the first quarter of 1985, we call them both 100. Because they look ahead ten years, the data in this graph ends at the beginning of 2001. We can see that movements in the red graph mirror movements in the blue graph fairly well, though not perfectly. (The correlation between the red and blue graphs is 0.49) If we believe that the introduction of inflation targeting in the fourth quarter of 1992 can be treated as a game-changing event, we can compare the right-hand end of the blue graph with the green graph instead – seeing that the mirroring becomes even better. (The break-adjusted series has a correlation of 0.83.)



**Figure 2: Comparison of normalised GDP series with quarterly growth (1985Q1 = 100)**



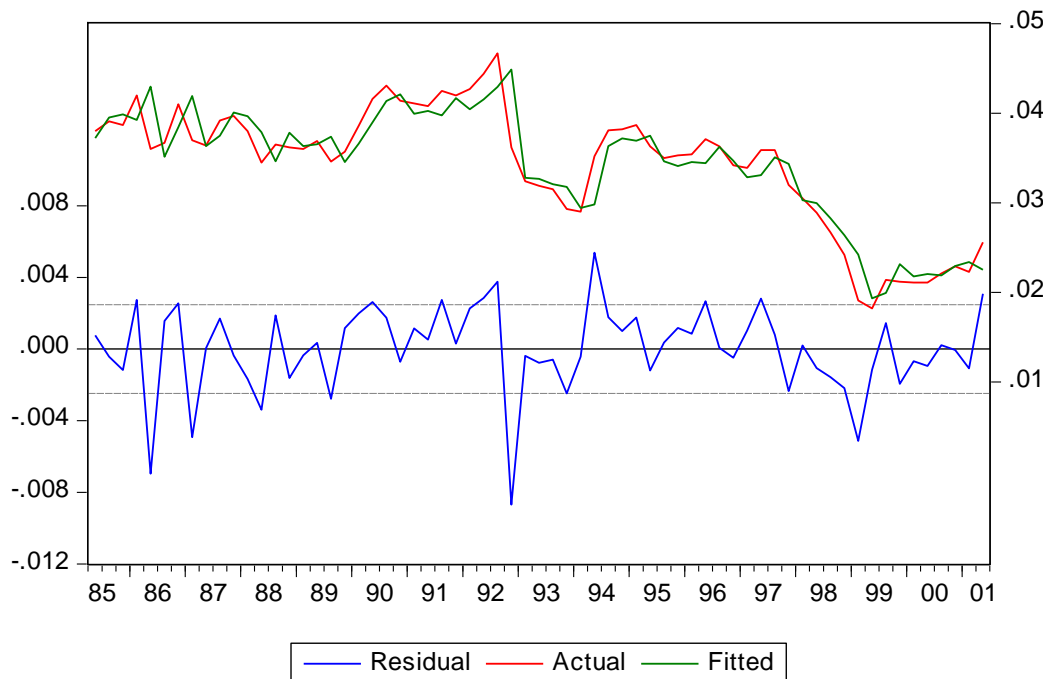
We first provide a simple analysis of the yields series, without discussing stationarity issues (which are discussed further below). Our gilt yield series can be modelled as an ARMA (1,1) process (with a declining trend, named “T” below).



**Table 1: ARMA (1,1) estimation results**

Dependent Variable: <b>YIELD</b>				
Method: Least Squares				
Sample (adjusted): 1985Q2 2001Q2				
Included observations: 65 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.044461	0.003579	12.42281	0.0000
T	-0.000292	8.72E-05	-3.351631	0.0014
AR(1)	0.746950	0.092764	8.052165	0.0000
MA(1)	0.336549	0.139958	2.404639	0.0192
R-squared	0.876829	Mean dependent var	0.034311	
Adjusted R-squared	0.870772	S.D. dependent var	0.006892	
S.E. of regression	0.002478	Akaike info criterion	-9.103483	
Sum squared resid	0.000374	Schwarz criterion	-8.969675	
Log likelihood	299.8632	Hannan-Quinn criter.	-9.050687	
F-statistic	144.7495	Durbin-Watson stat	2.049301	
Prob(F-statistic)	0.000000			
Inverted AR Roots	.75			
Inverted MA Roots	-.34			

**Figure 3: Fitted and residuals of values for the ARMA (1,1) series process**



The graph above indicates there 1992Q3 as a candidate date for a break in the series and indeed a Chow test confirms this suspicion.



**Table 2: Chow test on ARMA(1,1)**

<b>Chow Breakpoint Test: 1992Q3</b>			
Null Hypothesis: No breaks at specified breakpoints			
Equation Sample: 1985Q2 2001Q2			
F-statistic	8.680055	Prob. F(4,57)	0.0000
Log likelihood ratio	30.91995	Prob. Chi-Square(4)	0.0000
Wald Statistic	39.88169	Prob. Chi-Square(4)	0.0000

We have repeated the Chow test after integrating the gilt series (i.e. taking its first difference which should also resolve potential non-stationarity issues) and modelling it as an ARIMA (1,1).

**Table 3: ARIMA (1,1) estimation results**

<b>Dependent Variable: D(YIELD)</b>				
Method: Least Squares				
Sample (adjusted): 1985Q3 2001Q2				
Included observations: 64 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000189	0.000357	-0.530306	0.5978
AR(1)	-0.643664	0.186167	-3.457446	0.0010
MA(1)	0.844661	0.119575	7.063841	0.0000
R-squared	0.076295	Mean dependent var	-0.000194	
Adjusted R-squared	0.046010	S.D. dependent var	0.002607	
S.E. of regression	0.002547	Akaike info criterion	-9.062452	
Sum squared resid	0.000396	Schwarz criterion	-8.961254	
Log likelihood	292.9985	Hannan-Quinn criter.	-9.022585	
F-statistic	2.519216	Durbin-Watson stat	2.038245	
Prob(F-statistic)	0.088871			
Inverted AR Roots	-.64			
Inverted MA Roots	-.84			

Again, the Chow test confirms the presence of a structural break in the series in at 1992Q3.

**Table 4: Chow test on ARIMA(1,1)**

<b>Chow Breakpoint Test: 1992Q3</b>			
Null Hypothesis: No breaks at specified breakpoints			
Equation Sample: 1985Q3 2001Q2			
F-statistic	7.814110	Prob. F(3,58)	0.0002
Log likelihood ratio	21.72494	Prob. Chi-Square(3)	0.0001
Wald Statistic	53.73791	Prob. Chi-Square(3)	0.0000



### 3.4 Caveats

We focus on ten-year index-linked gilt yields and growth rates here. Five-year gilt yields can be significantly affected by policy expectations — e.g. in a recession policy interest rates may be set low, dragging down the five-year gilt yield. Since our data begins only in 1985, the use of twenty-year values would make our dataset very short (just five years instead of fifteen). However, we acknowledge that there is a compromise here. The actual growth rate could, in principle, deviate materially from the underlying sustainable growth rate even over a ten-year horizon. For example, one interpretation of our non-break-adjusted series could be that actual growth rates were below sustainable growth rates during the 1980s but then above sustainable growth rates during the 1990s (perhaps “catching up” on the “lost growth” of the 1980s). One implication of this reflection is that it is not obvious, despite the higher correlation, that our break-adjusted series is really the better series for correlating to ten-year-ahead growth rates.

Again, our proxy for the point sustainable growth rate is the average growth rate over a ten-year period, and thus quarterly movements in the growth series should be expected only imperfectly, and on average, to mirror movements in the yield series. If the point sustainable growth rate were, indeed, perfectly correlated with the yield series, then the average growth series we present would have something of the character of a rolling average of the sustainable growth series — it would, intrinsically, smooth it out.

It is therefore no surprise that, visually, it is clear the yields series in Figure 2 is notably more volatile than the growth series. Consequently it is thus likewise no surprise that models in differences (as opposed to the levels models we set out in the tables above) do not have statistically significant coefficients. The nature of the data series and the tests we conduct lend themselves more to levels models than to differences models.

### 3.5 Model

Our model explains movements in yields by a constant, the change in regime occurring in 1992Q3, and GDP, as set out in the following table.

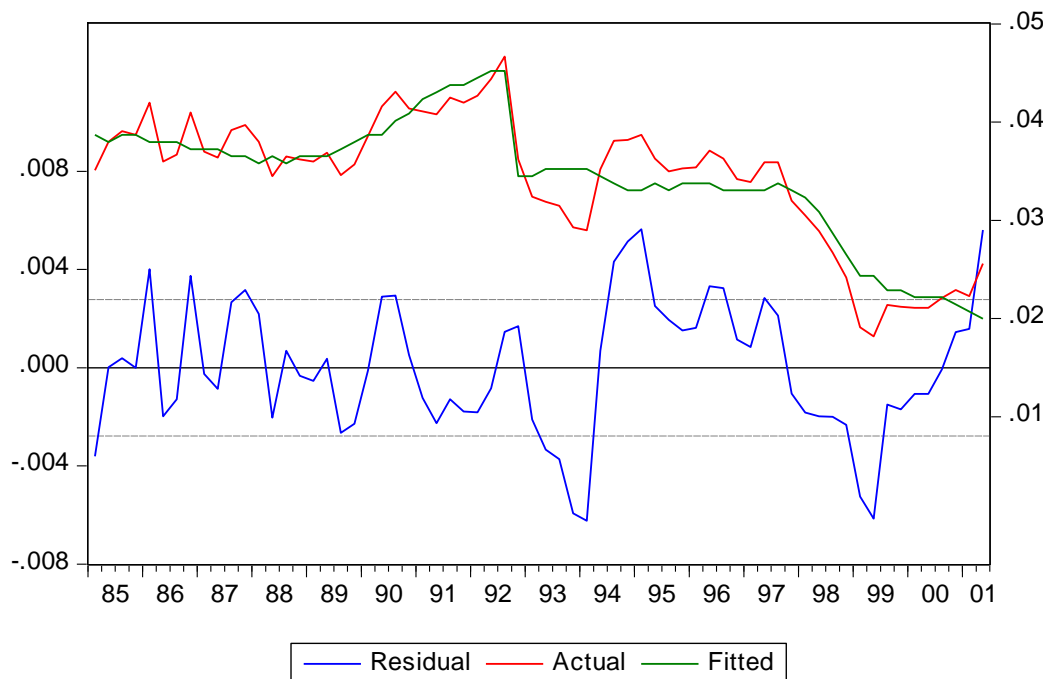
**Table 5: A Simple Model**

Dependent Variable: YIELD  
Method: Least Squares  
Sample: 1985Q1 2001Q2  
Included observations: 66  
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GDP	0.725478	0.095041	7.633279	0.0000
BREAK	-0.010731	0.000983	-10.91737	0.0000
C	0.020568	0.002454	8.380193	0.0000
R-squared	0.840297	Mean dependent var		0.034323
Adjusted R-squared	0.835227	S.D. dependent var		0.006840
S.E. of regression	0.002776	Akaike info criterion		-8.890969
Sum squared resid	0.000486	Schwarz criterion		-8.791439
Log likelihood	296.4020	Hannan-Quinn criter.		-8.851640
F-statistic	165.7408	Durbin-Watson stat		0.733382
Prob(F-statistic)	0.000000			



**Figure 4: Fitted and residuals of values for the model**



Thus, caveats notwithstanding, the upshot of our analysis is that the close relationship that theory predicts between the risk-free rate and the sustainable growth rate appears to be borne out in practice. The sustainable growth rate of the economy appears to have been fairly stable from the mid to late 1980s, risen somewhat in the early 1990s, and fallen fairly rapidly from the second quarter of 1997 to below its late 1980s trough.

A caveat/concern regarding the results of Table 5 is that if (as appears likely) the yields and GDP series are non-stationary, the model might be capturing a spurious relationship as opposed to a long-run equilibrium relationship. In fact Augmented Dickey Fuller (ADF) tests confirm that the yields and GDP series are  $I(1)$  (i.e. they series are non-stationary at the levels, but their first differences are stationary). Therefore, further analysis is required in order to test whether yields and GDP are cointegrated, in which case we can conclude that the model describes a long run equilibrium relationship.

We first notice that the Durbin-Watson (DW) statistics in Table 5 is materially different from zero, which is a first indication that the series might be cointegrated (if the regression were spurious we would expect a DW value close to zero). Furthermore, we have also performed an ADF test on the residuals of the regression of Table 5 (for a cointegrating relationship we would expect the residuals to be stationary). The resulting ADF test statistics of the residuals vary between -3.68 and -3.75 (depending on the version of the test performed, i.e. with/without trend and/or intercept). These values are larger than the asymptotic critical values (always at the 10 per cent confidence level, sometimes at 5 per cent) for residual-based unit root tests for cointegration, hence we can reject the hypothesis that the residuals are non-stationary.<sup>8</sup>

So, to summarise: We find that both the yield series and the GDP series are non-stationary (as is often the case for macroeconomic variables), but the residuals of the model correlating

<sup>8</sup> The values are -4.29, -3.74 and -3.45 respectively for 1%, 5% and 10% confidence, see Davidson, R. and J.G. McKinnon (1993), "Estimation and Inference in Econometrics", Oxford University Press.



the two series are stationary. So the possibility that the correlation we find is spurious is statistically rejected.

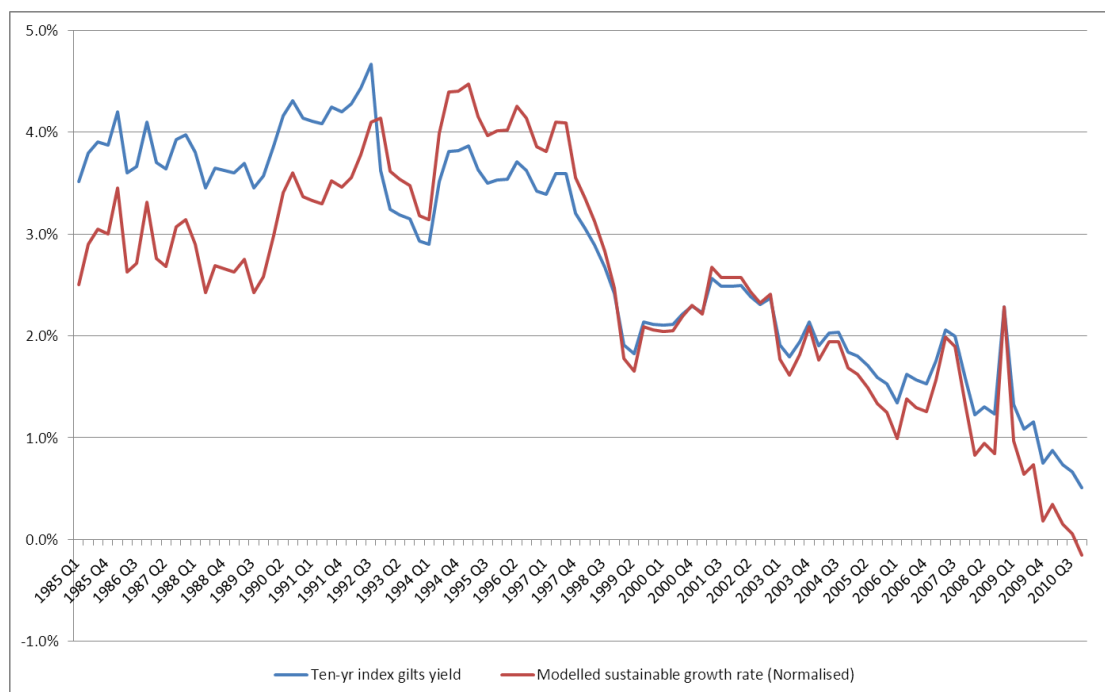
## 4 Use of model for predictions

In principle, then, we can read off the underlying sustainable growth of the economy over, say, the next ten years from the ten-year index-linked gilt yield. Conversely, if we have an alternative method of estimating growth rates further out (say, from five years ahead to fifteen years ahead), we could use such estimates to infer back to an expected level of index-linked gilt yields in five years' time.

It is of interest to illustrate how this might be done, particularly because the results are significantly at variance with the predictions of sustainable growth rates produced by cyclical analysis approaches such as those used by official UK government forecasting agencies such as the Office for Budget Responsibility or the Bank of England.

In the following graph, we apply our model. We note that in the model presented there, we assume that the sustainable growth rate in 1985Q1 is equal to the actual 10-year growth rate for the next ten years ahead (2.50 per cent, versus a value of 2.0 generated by the model itself). Changes in the level of yields from our model then constitute changes in the level of yields from this 2.50 per cent startpoint. The effect is that the levels in the model represented in the figure are around 0.5 per cent above those generated from the model in the table. For this reason the modelled sustainable growth rate in the figure is described as “Normalised”.

**Figure 1: Modelled sustainable growth rate versus gilts yield**



So, according to our model, the sustainable growth rate peaked at about 4 per cent in the mid-1990s, and had fallen to about 2 per cent by the end of 2000. The rate continues around 2 per cent until 2002, when it starts falling again. There is a brief odd blip up in mid-2007, and then the spike in late 2008 (which surely reflects a sudden rise in sovereign default risk – i.e. the



model is breaking down as the index-linked gilt yield is no longer nearly-risk-free). From the first quarter of 2009 we also get a downward distortion, as quantitative easing is estimated by the Bank of England to take perhaps a whole percentage point off yields.

Thus, perhaps the most recent index-linked gilt yields in our series are unreliable indicators of the risk-free rate. As an alternative measure of the risk-free rate, we can consider regulatory determinations, which in the UK use index-linked gilt yields as their main source of evidence, but seek to make adjustments for factors such as sovereign credit risk and quantitative easing.

By way of illustration of the potential significance of the difference between the approach sketched here and the standard approach deploying cyclical analysis, we observe the following. At the time of writing (October 2011), the most recent regulatory determination of the risk-free rate in the UK was Ofcom's 20 July 2011 WBA Charge Control Statement, where the risk-free rate was determined at 1.4 per cent. On our model, that would equate to a sustainable growth rate of 1.1 per cent. At the time of writing, the sustainable growth rate estimated by the Office for Budget Responsibility is 2.35 per cent — very significantly higher.<sup>9</sup>

## 5 Conclusions

In this paper, we have pointed out that theory implies there should be expected to be a relationship between the risk-free rate of return and the sustainable growth rate of economies. A natural proxy for the risk-free rate is the return on index-linked gilts, whilst a natural proxy for the sustainable growth rate is the average growth rate actually achieved, on average, over the forthcoming ten years. We have seen that movements in UK index-linked gilts can now be seen to have been highly correlated with movements in average GDP growth rates (a relationship that has not been obvious until the recent recession). As theory implies should be possible, we appear to be able to infer an excellent forecast for the ten-year ahead growth rate of the economy from the yield on index-linked bonds (once we have properly adjusted for distortions such as sovereign credit risk or quantitative easing).

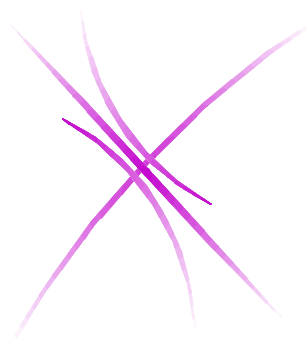
When we apply this model to current data, the forecast growth rate is markedly lower than that produced by cyclical-analysis based approaches such as those used by the UK Office for Budget Responsibility. The fact that such significant differences can emerge might suggest that statistical agencies should take account of index-linked government bonds data in formulating their estimates of sustainable growth rates and thence of output gaps.

*Andrew Lilico & Stefano Ficco, Europe Economics, October 2011*

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<sup>9</sup> Subsequent to the drafting of this paper, the OBR downgraded its estimate of the sustainable growth rate to 1 per cent since the recession began and 1.2 per cent in 2012 (rising to 2.3 per cent by 2014), in its November 2011 *Economic and Fiscal Outlook*, [http://cdn.budgetresponsibility.independent.gov.uk/Autumn2011EFO\\_web\\_version138469072346.pdf](http://cdn.budgetresponsibility.independent.gov.uk/Autumn2011EFO_web_version138469072346.pdf) (especially Section 3). See also, Pybus, T. *Estimating the UK's historical output gap*, Working Paper No. 1, Office for Budget Responsibility, November 2011, especially Table 3.1 in which the 2005-2010 estimate of the annual rate of growth in potential output falls to 1.3 per cent. This brings the OBR estimates very much in line with those presented here, further supporting the interest of the model.





# Europe Economics

Chancery House, 53-64 Chancery Lane, London WC2A 1QU,  
Tel: (+44) (0) 20 7831 4717, Fax: (+44) (0) 20 7831 4515  
[www.europe-economics.com](http://www.europe-economics.com)